Congestion due to drivers searching for parking: data-driven modeling and optimization

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Curbside parking in Seattle

Image credit: Ana Arevalo, CBS, Washington DC

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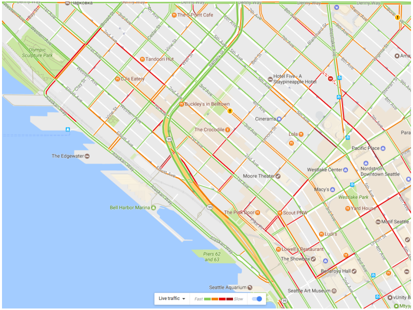
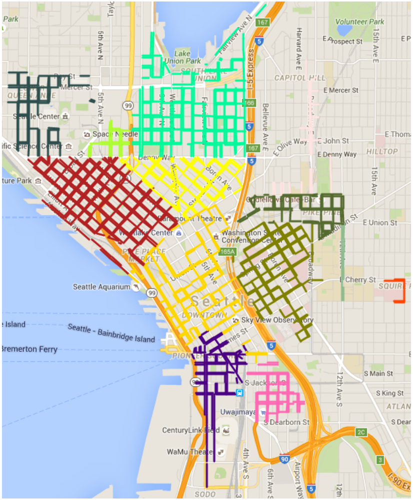
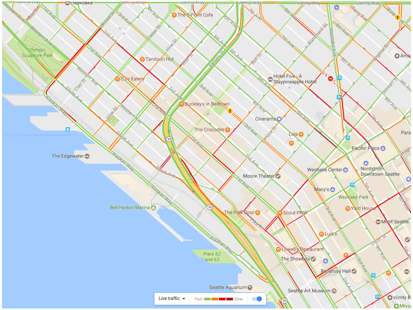


Estimated 30% of drivers on city streets searching for parking

Curbside parking in Seattle





Engineering Problem

If 30% of traffic is searching for parking, can we adapt parking strategies to mitigate costly congestion

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Solutions rely on empirical study and simulation to evaluate resource performance

Occupancy

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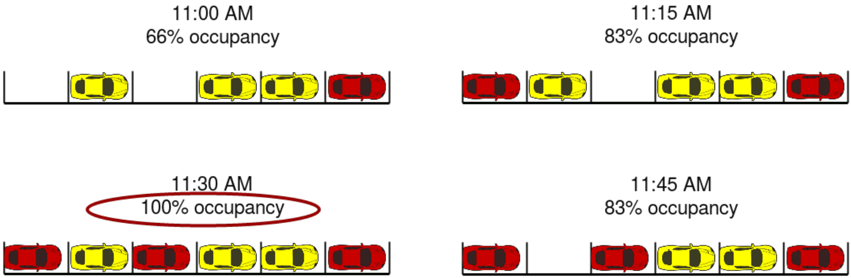
- Once required manual counting, can estimate with digital parking meters
- SDOT aims for a per-block-face occupancy level in the range of 75%—85% on an *hourly* basis
- Commonly accepted domain literature claims congestion occurs at 100% occupancy

Occupancy



83% hourly occupancy

Research Questions

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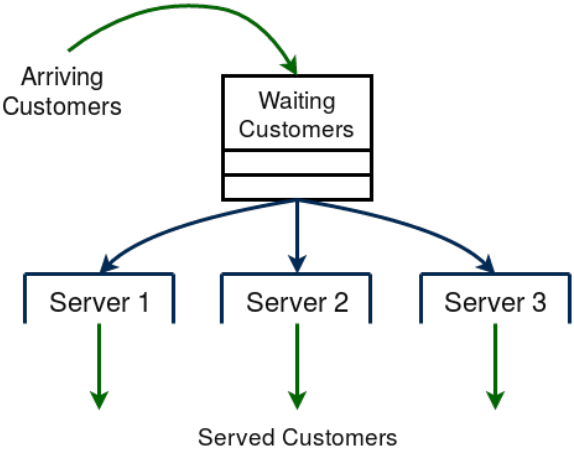
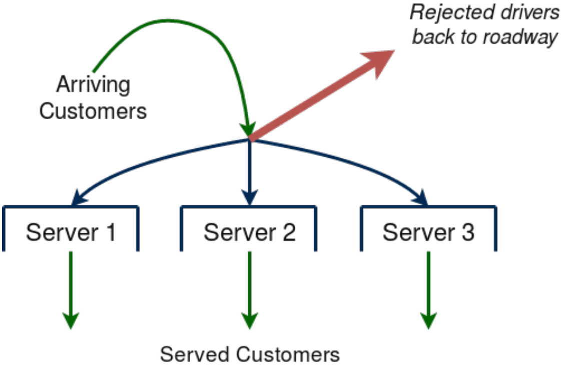
Let's model downtown curbside parking as a network of interdependent queues.

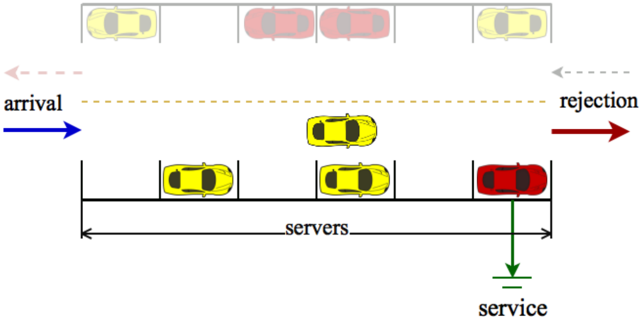
Multi-server Queue



Bufferless Multi-server Queue



Block-face as a Queue



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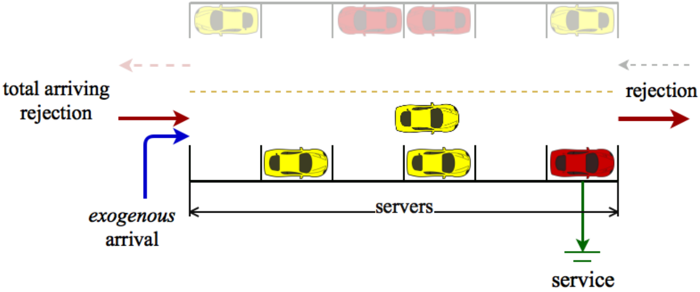
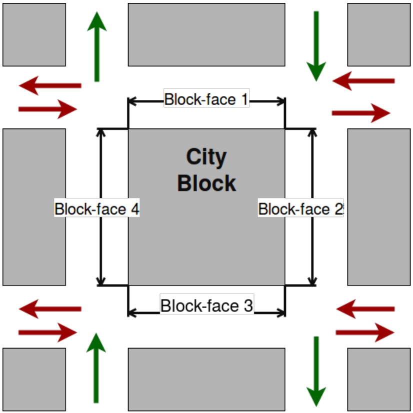
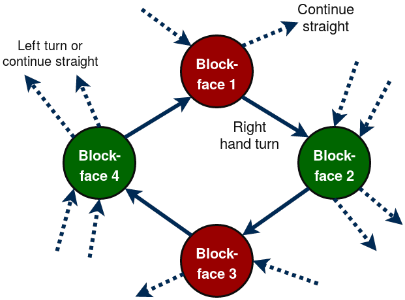
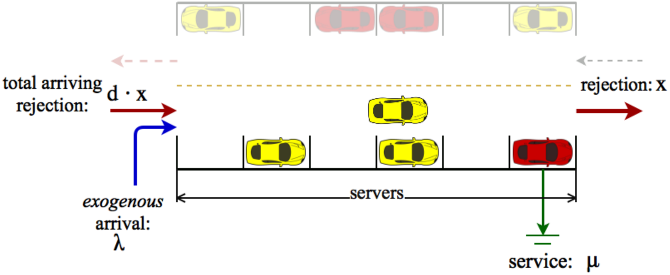


Block-face Queue Network



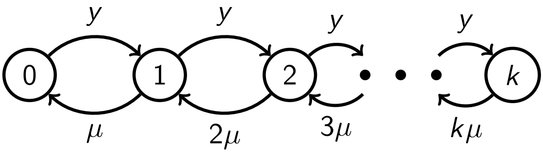


Properties of M/G/k/k Block-face Queue



There is some total arrival rate $y = \lambda + d \cdot x$ that depends on neighboring rejection rates

Properties of M/G/k/k Queue



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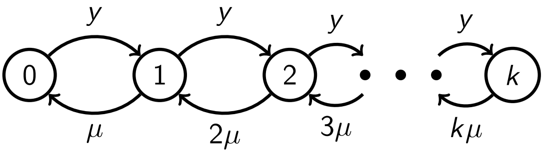

Stationary distribution solution to $\boldsymbol{\pi} Q = 0$

$$\boldsymbol{\pi} = \langle \pi_0, \pi_1, \cdots \pi_k \rangle, \qquad \pi_i = \pi_0^{-1} \cdot \frac{\left(\frac{y}{\mu}\right)^i}{i!}$$

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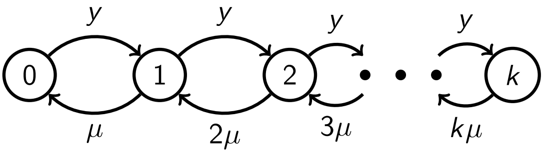

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Probability queue is full: π_k

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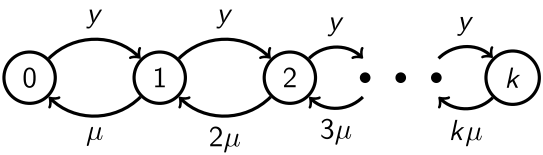

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Probability queue is full: $\pi_k \quad \rightarrow y \cdot \pi_k = x$

Results

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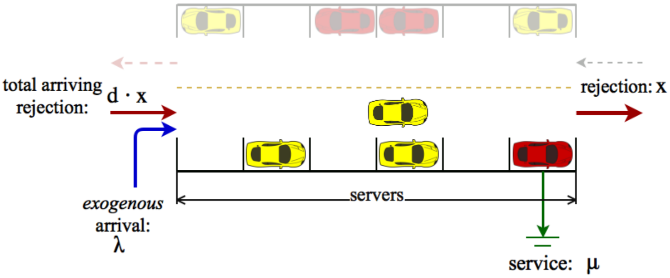
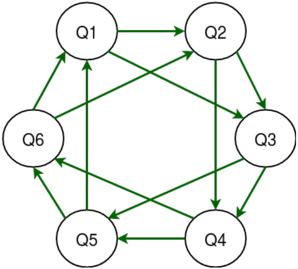
- We'll illustrate with a hypothetical optimization result
- And we'll conclude with discussion on future work

Block-face Queue Notation



Let $y = \lambda + d \cdot x$

Symmetric/Uniform Networks



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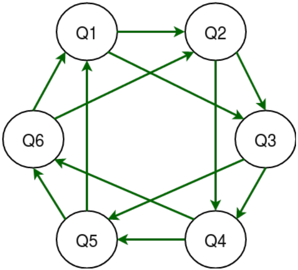

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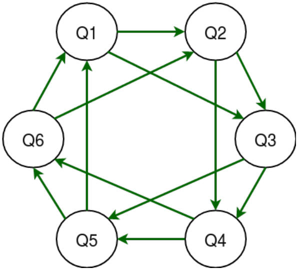

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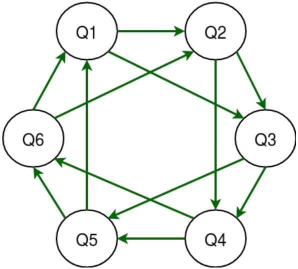
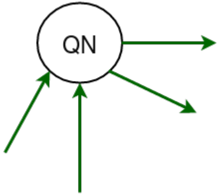


- Assume the graph is *d-regular*
- Assume uniform occupancy, service rate, number of servers
- Assume drivers search uniformly at random

If occupancy is uniform, then rejections are the same everywhere and we get a conservation equation:

$$y\pi_k = (\lambda + d \cdot x)\pi_k = d \cdot x \qquad (1)$$

Symmetric/Uniform Networks



$k+2$ equations;
$\boldsymbol{\pi}$, λ, x unknown

$$\boldsymbol{\pi} Q = 0 \tag{2a}$$

$$\sum_i \pi_i = 1 \tag{2b}$$

$$(\lambda + dx)\pi_k = dx \tag{2c}$$

Symmetric/Uniform Networks

(For simplicity, let $\mu = 1$) Rearranging (2c), and substituting formula for π_k in terms of π_0:

$$\frac{k-\lambda}{k!}y^k + \frac{(k-1)-\lambda}{(k-1)!}y^{k-1} + \cdots + (1-\lambda)y - \lambda = 0 \qquad (3)$$

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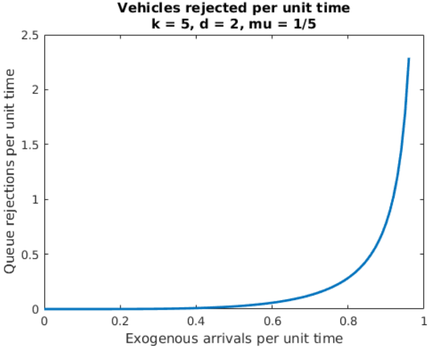
The sequence of sign changes undergoes only one sign change, so by Descartes' Rule of Signs, y is unique and positive. Further, by application of the IVT, $y > \lambda$

Arrival Rates



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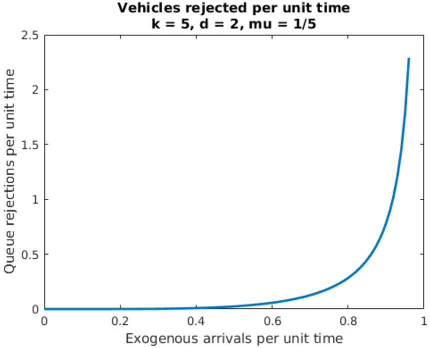
Arrival Rates



- Rejections asymptotic in arrivals
- Need way to get from occupancy data to arrival rate, and subsequent rejection rate—let's look at occupancy data

Non-uniform Networks: Belltown

Figure 1: A typical Monday at 11 AM in Belltown

Non-uniform Networks: Belltown

Invalid assumptions for Belltown:

- Uniform occupancy
- Network is d-regular
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Little's Law

In typical queueing problems, one designs a queue around expected arrival or service rates. We want to determine arrival rates *from* some occupancy level u.

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Little's Law is an expression for time average number of customers L in the system: $L = \gamma \cdot w$. Occupancy is simply normalized by number of servers k:

$$L = y\,(1 - \pi_k) \cdot \frac{1}{\mu} \tag{4}$$

$$u = \frac{y}{k\mu}\,(1 - \pi_k) \tag{5}$$

Little's Law

(Again let $\mu = 1$ for simplicity) Substituting formula for π_k in terms of π_0 into (5), and rearranging, we again get polynomial in y.

$$\frac{k - uk}{k!} y^k + \cdots (1 - uk) y - uk = 0 \tag{6}$$

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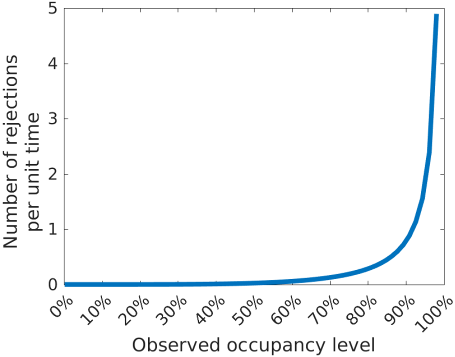
Note, this version relies on occupancy, not conservation equation.
Use SDOT occupancy data directly.

Occupancy to Congestion



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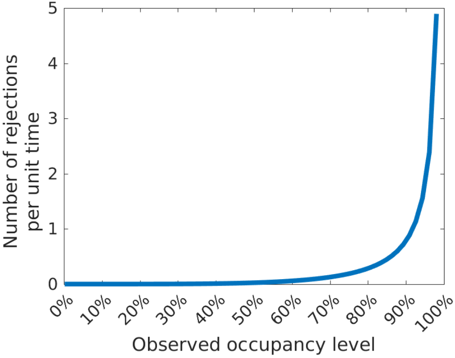
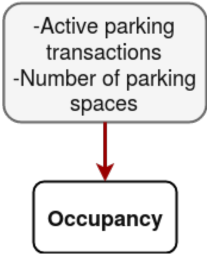


- Rejections asymptotic in occupancy
- Can estimate proportion of through-traffic in search of parking by calculating for rejection rates at each block-face.

Calculating Congestion from Data



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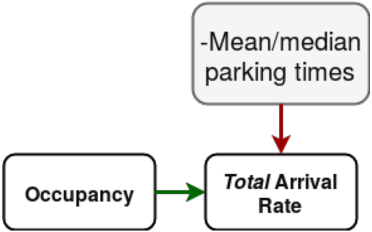

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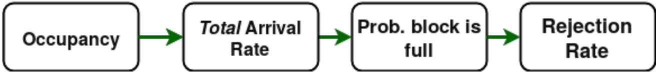
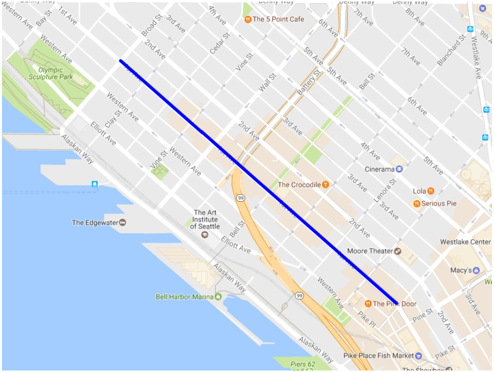
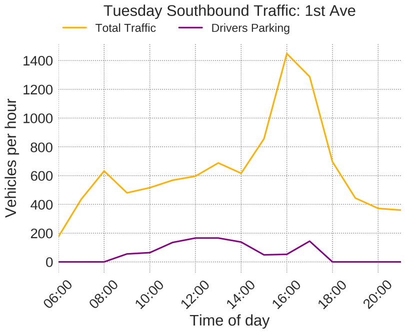


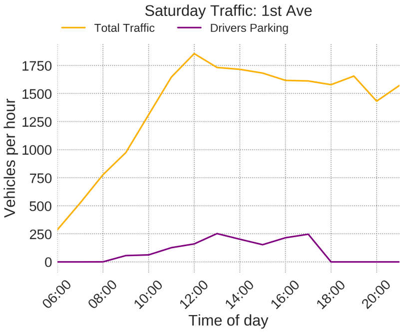
Proportion of Traffic Due to Parkers



We'll compare the total volume of rejections of block-faces along an arterial corridor to through-traffic volume data collected along the arterial.

Proportion of Traffic Due to Parkers





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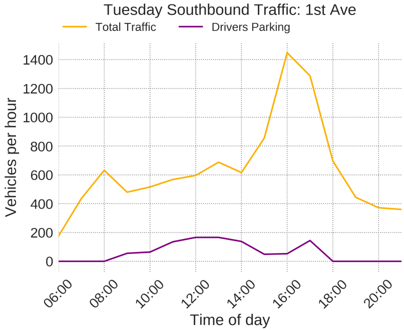

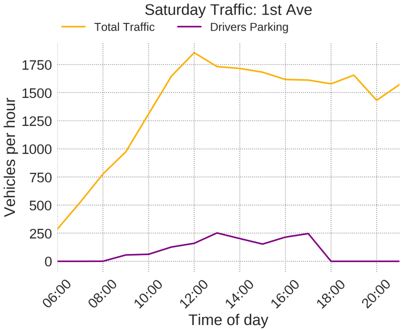

What is the time-delay impact to through-traffic?

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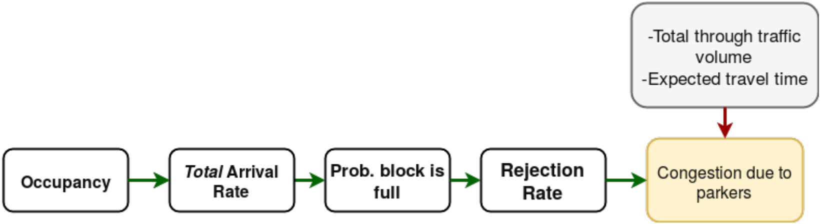


Congestion Caused by Parkers

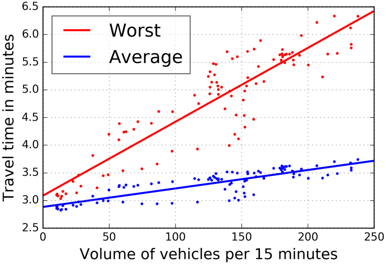


Figure 2: Estimates of travel time delay curve for measured volume versus historical delay

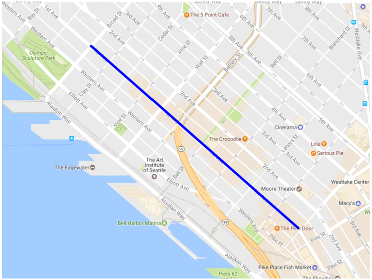


Figure 3: Belltown arterials with SDOT traffic volume data

Congestion Caused by Parkers

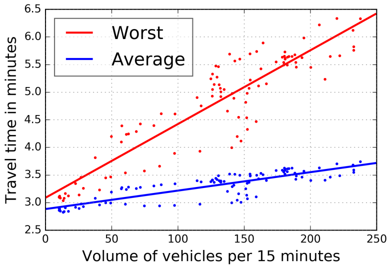


Figure 4: Estimates of travel time delay curve for measured volume versus historical delay

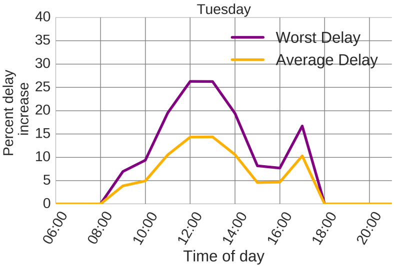
T : volume of cars $\rightarrow$ expected delay

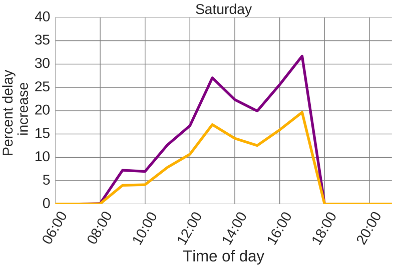
Percent increase in delay:

$$\frac{T(N_{\text{total}})}{T(N_{\text{total}} - N_{\text{parking}})} - 1$$

Congestion Caused by Parkers

Average percent increase to delay





Congestion Optimization

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- We can take an observed occupancy level to a resulting level of congestion
- Cities are already developing parking control policies to minimize impact to congestion: e.g. time of day or locational pricing
- Can we describe an optimization program that minimizes the impact to congestion?

Congestion Optimization

- Price is among our only control variables
- Design an optimal parking policy with congestion as specified constraints—evening parking congestion may be acceptable while rush-hour parking congestion may not.

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$$
\begin{aligned}
\underset{\boldsymbol{p}}{\text{maximize}} \quad & \text{Occupancy}(\boldsymbol{p}) \\
\text{subject to} \quad & \text{congestion along road } i, \quad i = 1, \dots, m \\
& g_i(p_i) \leq \bar{x}_i
\end{aligned}
\tag{P-1}
$$

Objective: Occupancy as Price

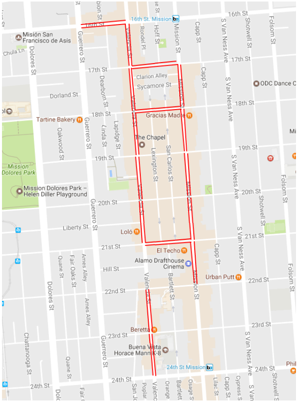

Figure 5: Curbside parking data in the Mission District of SF

- Price elasticity estimates from SFPark pilot study and companion 2013 study

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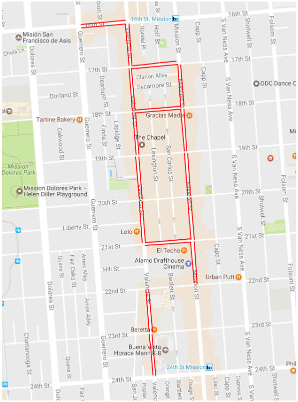

Figure 5: Curbside parking data in the Mission District of SF

- Price elasticity estimates from SFPark pilot study and companion 2013 study
- Use a linear price elasticity function $\mathcal{U} = 1 - \alpha p$

Constraints: Congestion $g(p)$

- Constraint values x_i depend on an implicit mapping based on eqn. (6) (Little's Law substitution for arrival rate)

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$$\mathcal{U}(p_i) = u_i \tag{7}$$

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- Let $f : u \to y$, the mapping takes an occupancy u to the unique arrival rate y

$$f(\mathcal{U}(p_i)) = y_i \tag{8}$$

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$$f(\mathcal{U}(p_i)) \cdot \pi_k = x_i \tag{9}$$

Constraints: Congestion $g(p)$

- Constraint values x_i depend on an implicit mapping based on eqn. (6) (Little's Law substitution for arrival rate)
- Let $f : u \rightarrow y$, the mapping takes an occupancy u to the unique arrival rate y

$$g_i(p_i) := f(\mathcal{U}(p_i)) \cdot \pi_k = x_i \tag{10}$$

Convexity of f

If we can show f is convex, we can find a unique solution (P-1) with gradient descent. Eqn. 6 written implicity:

$$F(y,u) = \frac{k - uk}{k!}y^k + \cdots (1 - uk)y - uk \tag{11}$$

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- By the implicit function theorem, (6) is continuously differentiable, can write $\frac{d^k y}{du^k}$ explicitly.
- Twice implicit differentiation gives $\frac{d^2 y}{du^2} \geq 0$. Then using Gauss-Lucas $\frac{dy}{du} > 0$, so we have f is convex (proof sketch in supplemental slides)

Price Control in Mission District

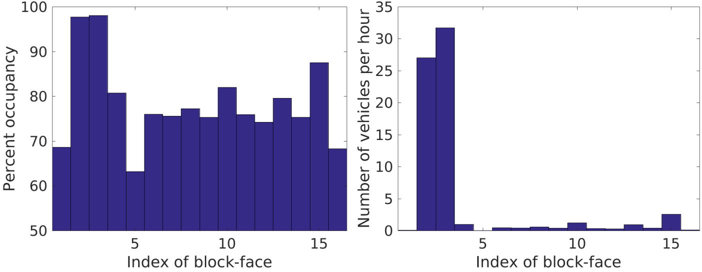
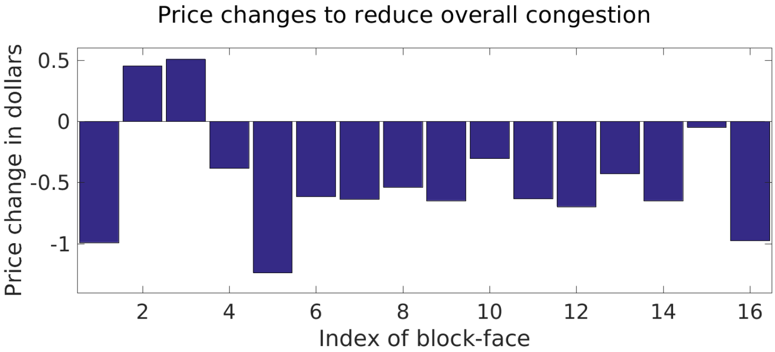


Figure 6: Noon weekday occupancy levels and resulting traffic estimates for Mission District, SF

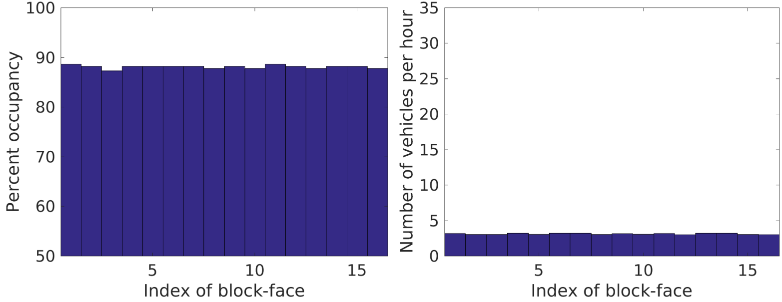
Price Control in Mission District

Noon weekday price changes to reduce rate of searching vehicles to no more than 1 per 12 minutes: Mission District, SF



Price Control in Mission District

Noon weekday controlled occupancy levels and resulting traffic estimates for Mission District, SF



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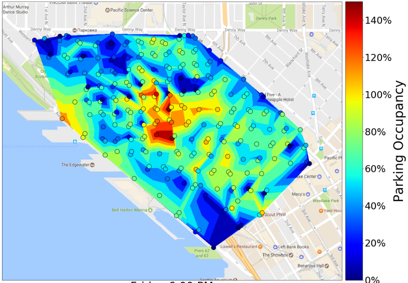
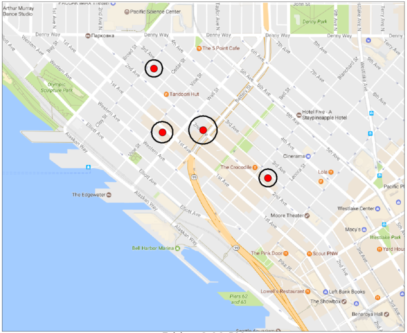


Friday 6:00 PM



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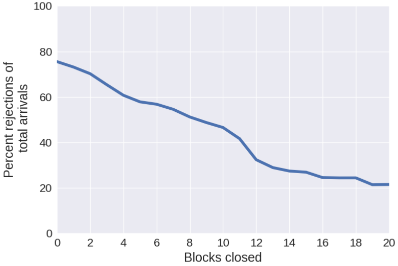

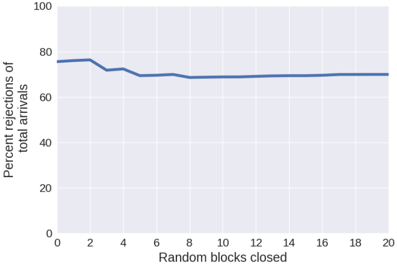

Discussion

What *are* we answering?

- Sharpen the "30% of traffic" estimate: depends on time of day *and* location
- Parking policy can be more rigorously designed with respect to end goal of controlling congestion

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What are we *not* answering?

- *Not* pricing against congestion due to individual drivers parking maneuvers
- Analyzing parking performance on a moment to moment basis, we're assuming the system can achieve equilibrium

Assumptions

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- Transaction data is representative of occupancy

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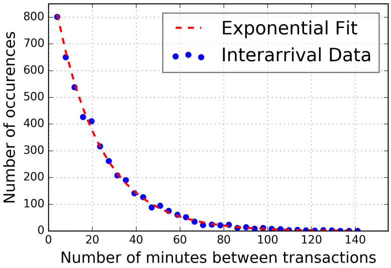

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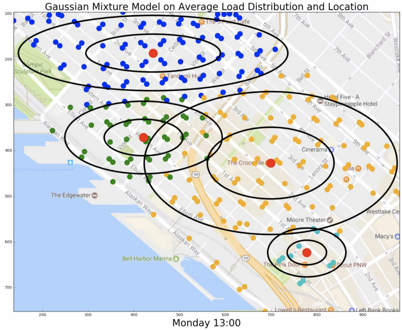
- Emerging effect of ride-sharing services—how will future curbside parking resources be most effectively utilized?

Future Work

How we're tackling these problems:

- Building a *structural* model *around* data that's currently available.
- Aiming to enable socially and politically actionable solutions to congestion



Credit: Tanner Fiez, UW EE

Concluding Remarks

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- We want to combine structural models from which control policy can be evaluated, with the naive data-analysis benefits of ML

Conclusion

Questions?

Data Sources

Data: IDAX, Seattle Dept of Transportation and `data.seattle.gov`

- block-face latitude/longitudes
- spaces per block (number of servers)
- curbside parking transactions since 2012 at each block-face (service times)
- traffic volume by time of day on select arterials (superset of drivers parking)

SDOT Data

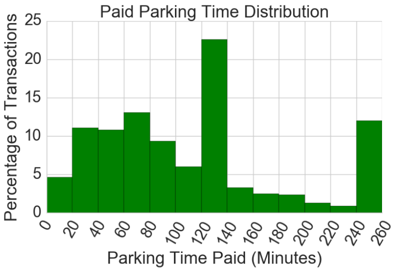

Figure 7: Distribution of transactions by paid parking time.

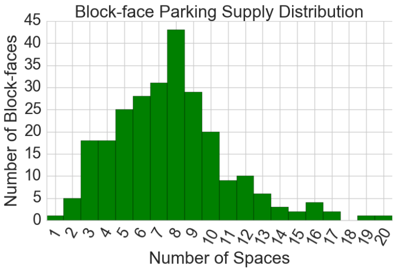

Figure 8: Distribution of parking spaces per block-face in Belltown.

Proof Sketch: Convexity of f

Let $x = ku$. Then we can think of (6) as

$$F(y,x) = (\tfrac{x}{k!} - \tfrac{1}{(k-1)!})y^k + \cdots + (\tfrac{x}{2!} - 1)y^2 + (x-1)y + x \quad (12)$$

$$y' = -D_x F \cdot (D_y F)^{-1} \quad (13)$$

and, by Quotient Rule:

$$y'' = \frac{D_x F \cdot (D_y^2 F \cdot y' + D_{x,y} F) - D_y F \cdot D_{y,x} F \cdot y'}{(D_y F)^2} \quad (14)$$

Proof Sketch: Convexity of f

Substituting in y' for the mixed partials, showing y'' boils down to showing

$$D_y^2 F \cdot y' + 2D_{y,x}F \geq 0 \tag{15}$$

Relying on the fact that (x, y) are a pair such that $F(x, y) = 0$, we get that

$$D_y^2 F \cdot y' + 2D_{y,x}F \geq y' F(x, y) = 0 \tag{16}$$

Proof Sketch: Convexity of f

We still need to show $y' > 0$.
By Gauss-Lucas (the roots of a polynomial are contained in the convex hull of the roots of its derivative), for fixed x all real parts of the roots of $D_y F$ are less than the root of $F(x, y)$. Since $D_y F \to -\infty$ as $y \to \infty$, at $F(x, y) = 0$. Recall we have that:

$$y' = -D_x F \cdot (D_y F)^{-1} \tag{17}$$

Since $D_y F \leq 0$ and since $D_x F > 0$, $y' > 0$