Congestion due to drivers searching for parking: data-driven modeling and optimization

Chase P Dowling
Tanner Fiez, Lillian J Ratliff, Baosen Zhang

University of Washington, Department of Electrical Engineering

December 6, 2017

Curbside parking in Seattle

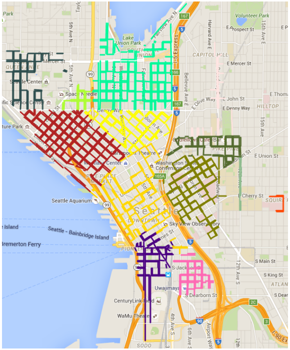


Image credit: Ana Arevalo, CBS, Washington DC

Estimated 30% of drivers on city streets searching for parking[1]

[1] Inci, Eren. "A review of the economics of parking." Economics of Transportation 4.1 (2015): 50-63.

Engineering Problem

If 30% of traffic is searching for parking, can we adapt parking strategies to mitigate costly congestion

Engineering Problem

If 30% of traffic is searching for parking, can we adapt parking strategies to mitigate costly congestion
Current solutions:

- Annual studies of parking resource performance

Engineering Problem

If 30% of traffic is searching for parking, can we adapt parking strategies to mitigate costly congestion
Current solutions:

- Annual studies of parking resource performance
- Demand-based pricing

Engineering Problem

If 30% of traffic is searching for parking, can we adapt parking strategies to mitigate costly congestion

Current solutions:

- Annual studies of parking resource performance
- Demand-based pricing
- Mobile Applications: SFPark, Go Mobile PGH

Engineering Problem

If 30% of traffic is searching for parking, can we adapt parking strategies to mitigate costly congestion

Current solutions:

- Annual studies of parking resource performance
- Demand-based pricing
- Mobile Applications: SFPark, Go Mobile PGH
- Places like Oslo banning cars from city center or Barcelona programmatically eliminating within certain sections

Engineering Problem

If 30% of traffic is searching for parking, can we adapt parking strategies to mitigate costly congestion

Current solutions:

- Annual studies of parking resource performance
- Demand-based pricing
- Mobile Applications: SFPark, Go Mobile PGH
- Places like Oslo banning cars from city center or Barcelona programmatically eliminating within certain sections

Solutions rely on empirical study and simulation to evaluate resource performance

Occupancy

- How does the city measure parking resource performance?

$$\text{Occupancy: } u = \frac{\#\text{ cars parked}}{\#\text{ parking spaces}}$$

Occupancy

- How does the city measure parking resource performance?

$$\text{Occupancy: } u = \frac{\#\text{ cars parked}}{\#\text{ parking spaces}}$$

- Once required manual counting, can estimate with digital parking meters

Occupancy

- How does the city measure parking resource performance?

$$\text{Occupancy: } u = \frac{\#\text{ cars parked}}{\#\text{ parking spaces}}$$

- Once required manual counting, can estimate with digital parking meters
- SDOT aims for a per-block-face occupancy level in the range of 75%—85% on an *hourly* basis

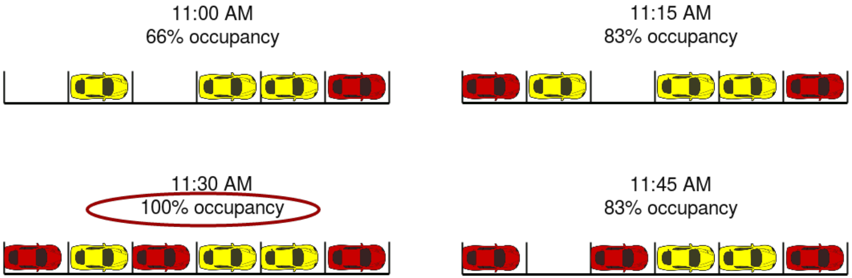
Occupancy

- How does the city measure parking resource performance?

$$\text{Occupancy: } u = \frac{\#\text{ cars parked}}{\#\text{ parking spaces}}$$

- Once required manual counting, can estimate with digital parking meters
- SDOT aims for a per-block-face occupancy level in the range of 75%—85% on an *hourly* basis
- Commonly accepted domain literature claims congestion occurs at 100% occupancy

Occupancy



83% hourly occupancy

Research Questions

Can we determine the amount of congestion drivers searching for parking are responsible for?

Research Questions

Can we determine the amount of congestion drivers searching for parking are responsible for?

If so, can we minimize the impact of this congestion while maintaining high occupancy?

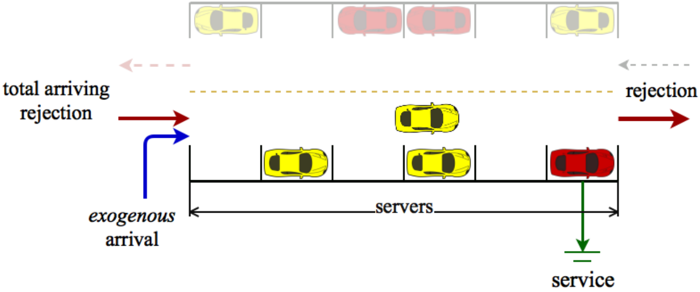
Research Questions

Can we determine the amount of congestion drivers searching for parking are responsible for?

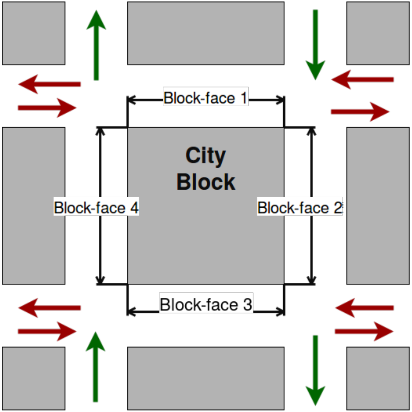
If so, can we minimize the impact of this congestion while maintaining high occupancy?

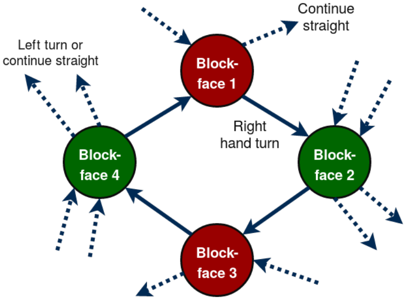
Let's model downtown curbside parking as a network of interdependent queues.

Block-face as a Queue

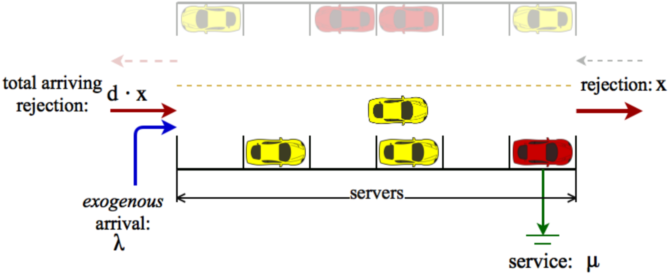


Block-face Queue Network



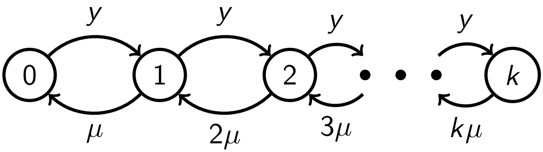


Properties of M/G/k/k Block-face Queue

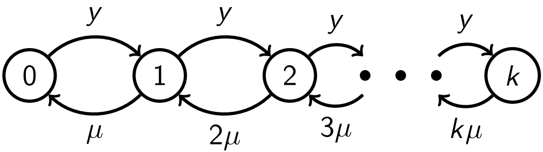


There is some total arrival rate $y = \lambda + d \cdot x$ that depends on neighboring rejection rates

Properties of M/G/k/k Queue



Properties of M/G/k/k Queue



Stationary distribution solution to $\boldsymbol{\pi} Q = 0$

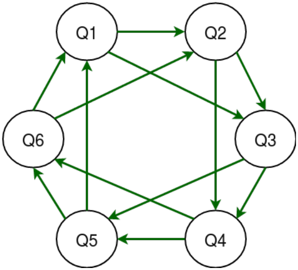
$$\boldsymbol{\pi} = \langle \pi_0, \pi_1, \cdots \pi_k \rangle, \quad \pi_i = \pi_0^{-1} \cdot \frac{\left(\frac{y}{\mu}\right)^i}{i!}$$

Probability queue is full: $\pi_k \quad \rightarrow y \cdot \pi_k = x$

Results

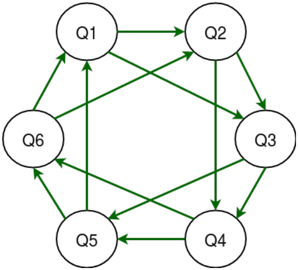
- First we'll gain some intuition in perfectly uniform networks
- We'll then analyze a real downtown network
- Then we'll state an optimization problem to minimize congestion
- We'll illustrate with a hypothetical optimization result
- And we'll conclude with discussion on future work

Symmetric/Uniform Networks



- Assume the graph is *d-regular*
- Assume uniform occupancy, service rate, number of servers
- Assume drivers search uniformly at random

Symmetric/Uniform Networks

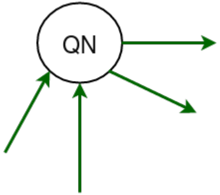


- Assume the graph is *d-regular*
- Assume uniform occupancy, service rate, number of servers
- Assume drivers search uniformly at random

If occupancy is uniform, then rejections are the same everywhere and we get a conservation equation:

$$y\pi_k = (\lambda + d \cdot x)\pi_k = d \cdot x \quad (1)$$

Symmetric/Uniform Networks



$k+2$ equations;
$\boldsymbol{\pi}$, λ, x unknown

$$\boldsymbol{\pi} Q = 0 \tag{2a}$$

$$\sum_i \pi_i = 1 \tag{2b}$$

$$(\lambda + dx)\pi_k = dx \tag{2c}$$

Symmetric/Uniform Networks

(For simplicity, let $\mu = 1$) Rearranging (2c), and substituting formula for π_k in terms of π_0:

$$\frac{k-\lambda}{k!}y^k + \frac{(k-1)-\lambda}{(k-1)!}y^{k-1} + \cdots + (1-\lambda)y - \lambda = 0 \qquad (3)$$

Symmetric/Uniform Networks

(For simplicity, let $\mu = 1$) Rearranging (2c), and substituting formula for π_k in terms of π_0:

$$\frac{k-\lambda}{k!}y^k + \frac{(k-1)-\lambda}{(k-1)!}y^{k-1} + \cdots + (1-\lambda)y - \lambda = 0 \quad (3)$$

The sequence of sign changes undergoes only one sign change, so by Descartes' Rule of Signs, y is unique and positive. Further, by application of the IVT, $y > \lambda$

Non-uniform Networks: Belltown

Figure 1: A typical Monday at 11 AM in Belltown

Non-uniform Networks: Belltown

Invalid assumptions for Belltown:

- Uniform occupancy
- Network is d-regular
- Uniform number of servers

Non-uniform Networks: Belltown

Invalid assumptions for Belltown:

- Uniform occupancy
- Network is d-regular
- Uniform number of servers

Model assumptions we make:

- Drivers exhibit a uniform search strategy
- Adjacent blocks see similar occupancy levels as a result of rejections from neighbors

Little's Law

In typical queueing problems, one designs a queue around expected arrival or service rates. We want to determine arrival rates *from* some occupancy level u.

Little's Law is an expression for time average number of customers L in the system: $L = \gamma \cdot w$.

Little's Law

In typical queueing problems, one designs a queue around expected arrival or service rates. We want to determine arrival rates *from* some occupancy level u.

Little's Law is an expression for time average number of customers L in the system: $L = \gamma \cdot w$. Occupancy is simply normalized by number of servers k:

$$L = y\left(1 - \pi_k\right) \cdot \frac{1}{\mu} \tag{4}$$

$$u = \frac{y}{k\mu}\left(1 - \pi_k\right) \tag{5}$$

Little's Law

(Again let $\mu = 1$ for simplicity) Substituting formula for π_k in terms of π_0 into (5), and rearranging, we again get polynomial in y.

$$\frac{k - uk}{k!} y^k + \cdots (1 - uk) y - uk = 0 \tag{6}$$

Little's Law

(Again let $\mu = 1$ for simplicity) Substituting formula for π_k in terms of π_0 into (5), and rearranging, we again get polynomial in y.

$$\frac{k-uk}{k!}y^k + \cdots (1-uk)y - uk = 0 \tag{6}$$

By similar application of Descartes' Rule of Signs, y is unique and positive for $u \in [0, 1)$.

Little's Law

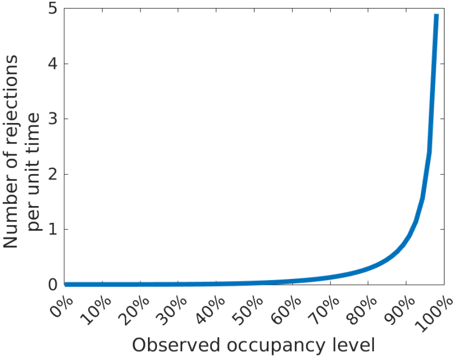
(Again let $\mu = 1$ for simplicity) Substituting formula for π_k in terms of π_0 into (5), and rearranging, we again get polynomial in y.

$$\frac{k - uk}{k!} y^k + \cdots (1 - uk)y - uk = 0 \tag{6}$$

By similar application of Descartes' Rule of Signs, y is unique and positive for $u \in [0, 1)$.

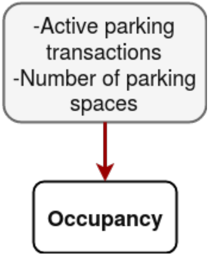
Note, this version relies on occupancy, not conservation equation. Use SDOT occupancy data directly.

Occupancy to Congestion

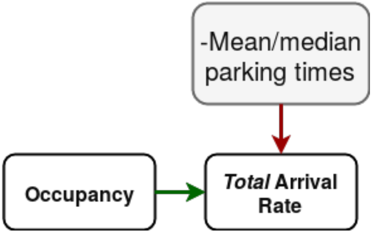


- Rejections asymptotic in occupancy
- Can estimate proportion of through-traffic in search of parking by calculating for rejection rates at each block-face.

Calculating Congestion from Data

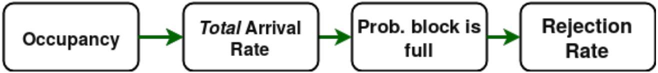


Calculating Congestion from Data

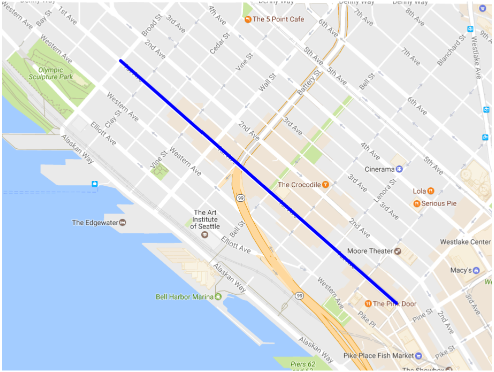


Calculating Congestion from Data

Calculating Congestion from Data



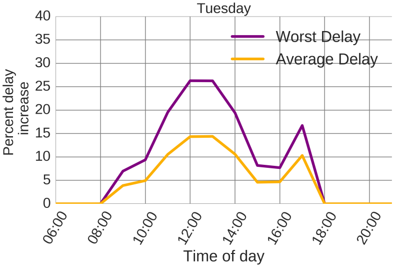
Proportion of Traffic Due to Parkers

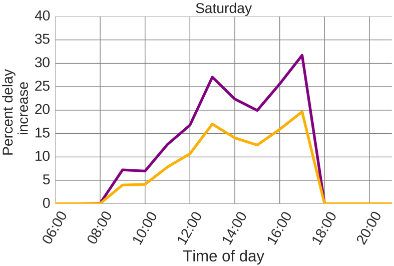


We'll compare the total volume of rejections of block-faces along an arterial corridor to through-traffic volume data collected along the arterial.

Congestion Caused by Parkers

With linear time delay model. Further details in proceedings.
Average percent increase to delay on 1st Ave. in Belltown:





Congestion Optimization

- We can take an observed occupancy level to a resulting level of congestion

Congestion Optimization

- We can take an observed occupancy level to a resulting level of congestion
- Cities are already developing parking control policies to minimize impact to congestion: e.g. time of day or locational pricing

Congestion Optimization

- We can take an observed occupancy level to a resulting level of congestion
- Cities are already developing parking control policies to minimize impact to congestion: e.g. time of day or locational pricing
- Can we describe an optimization program that minimizes the impact to congestion?

Congestion Optimization

- Price is among our only control variables
- Design an optimal parking policy with congestion as specified constraints—evening parking congestion may be acceptable while rush-hour parking congestion may not.

Congestion Optimization

- Price is among our only control variables
- Design an optimal parking policy with congestion as specified constraints—evening parking congestion may be acceptable while rush-hour parking congestion may not.

$$\begin{aligned} \underset{\boldsymbol{p}}{\text{maximize}} \quad & \text{Occupancy}(\boldsymbol{p}) \\ \text{subject to} \quad & \text{congestion along road } i, \quad i = 1, \ldots, m \\ & g_i(p_i) \leq \bar{x}_i \end{aligned} \tag{P-1}$$

Objective: Occupancy as Price

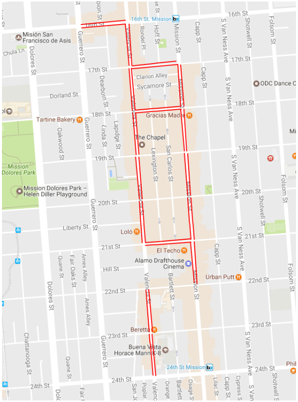


Figure 2: Curbside parking data in the Mission District of SF

- Price elasticity estimates from SFPark pilot study and companion 2013 study
- Use a linear price elasticity function $\mathcal{U} = 1 - \alpha p$

Constraints: Congestion $g(p)$

- Constraint values x_i depend on an implicit mapping based on eqn. (6) (Little's Law substitution for arrival rate)

Constraints: Congestion $g(p)$

- Constraint values x_i depend on an implicit mapping based on eqn. (6) (Little's Law substitution for arrival rate)

$$\mathcal{U}(p_i) = u_i \tag{7}$$

Constraints: Congestion $g(p)$

- Constraint values x_i depend on an implicit mapping based on eqn. (6) (Little's Law substitution for arrival rate)
- Let $f : u \to y$, the mapping takes an occupancy u to the unique arrival rate y

$$f(\mathcal{U}(p_i)) = y_i \tag{8}$$

Constraints: Congestion $g(p)$

- Constraint values x_i depend on an implicit mapping based on eqn. (6) (Little's Law substitution for arrival rate)
- Let $f : u \to y$, the mapping takes an occupancy u to the unique arrival rate y

$$f(\mathcal{U}(p_i)) \cdot \pi_k = x_i \tag{9}$$

Constraints: Congestion $g(p)$

- Constraint values x_i depend on an implicit mapping based on eqn. (6) (Little's Law substitution for arrival rate)
- Let $f : u \rightarrow y$, the mapping takes an occupancy u to the unique arrival rate y

$$g_i(p_i) := f(\mathcal{U}(p_i)) \cdot \pi_k = x_i \tag{10}$$

Convexity of f

If we can show f is convex, we can find a unique solution (P-1) with gradient descent. Eqn. 6 written implicity:

$$F(y,u) = \frac{k - uk}{k!} y^k + \cdots (1 - uk) y - uk \tag{11}$$

Convexity of f

If we can show f is convex, we can find a unique solution (P-1) with gradient descent. Eqn. 6 written implicity:

$$F(y,u) = \frac{k - uk}{k!} y^k + \cdots (1 - uk)y - uk \tag{11}$$

- By the implicit function theorem, (6) is continuously differentiable, can write $\frac{d^k y}{du^k}$ explicitly.
- Twice implicit differentiation gives $\frac{d^2 y}{du^2} \geq 0$. Then using Gauss-Lucas $\frac{dy}{du} > 0$, so we have f is convex (proof sketch in supplemental slides)

Price Control in Mission District

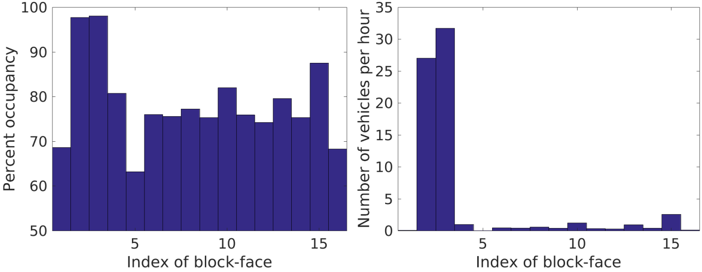
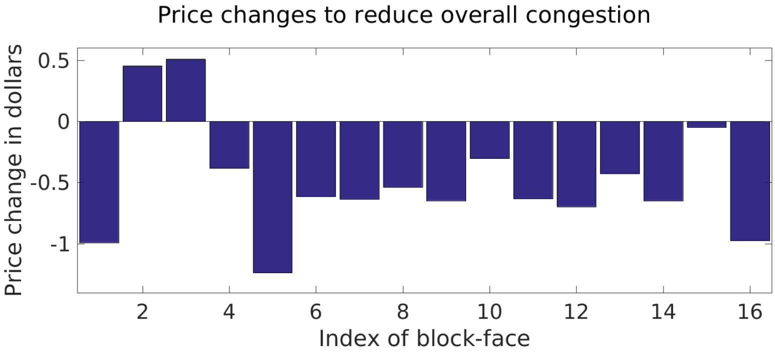


Figure 3: Noon weekday occupancy levels and resulting traffic estimates for Mission District, SF

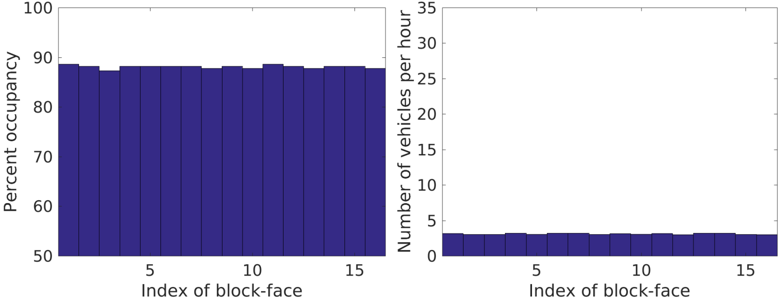
Price Control in Mission District

Noon weekday price changes to reduce rate of searching vehicles to no more than 1 per 12 minutes: Mission District, SF



Price Control in Mission District

Noon weekday controlled occupancy levels and resulting traffic estimates for Mission District, SF



Control Without Accurate Estimates of Price Elasticity

State of the art estimates of price elasticity are not necessarily concave. Evaluate the limiting case of $p \to \infty$

Control Without Accurate Estimates of Price Elasticity

State of the art estimates of price elasticity are not necessarily concave. Evaluate the limiting case of $p \to \infty$

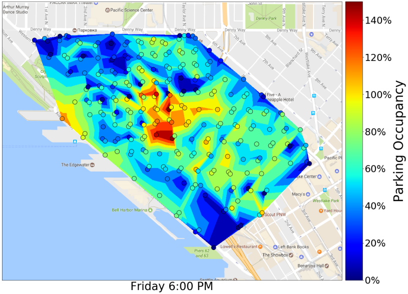


Figure 4: Contour plot of historical occupancy

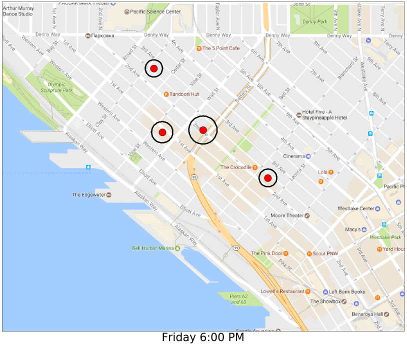


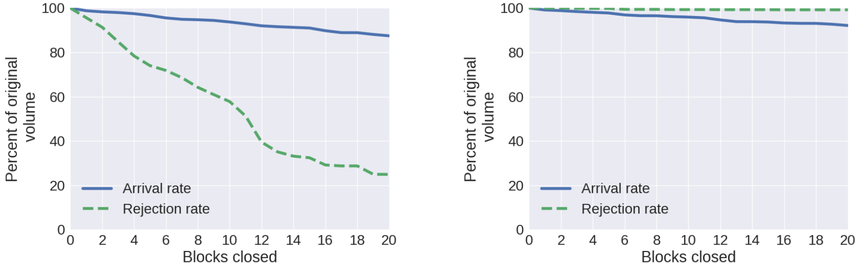
Figure 5: Clustered GMM centroids

Control Without Accurate Estimates of Price Elasticity

Closing highest occupancy blocks versus closing random choices yields largest impact on net-work wide rejections as a proportion of total arrivals.

Control Without Accurate Estimates of Price Elasticity

Closing highest occupancy blocks versus closing random choices yields largest impact on net-work wide rejections as a proportion of total arrivals.



Discussion

What *are* we answering?

- Sharpen the "30% of traffic" estimate: depends on time of day *and* location
- Parking policy can be more rigorously designed with respect to end goal of controlling congestion

Discussion

What *are* we answering?

- Sharpen the "30% of traffic" estimate: depends on time of day *and* location
- Parking policy can be more rigorously designed with respect to end goal of controlling congestion

What are we *not* answering?

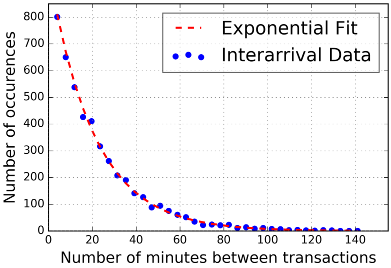
- *Not* pricing against congestion due to individual drivers parking maneuvers
- Analyzing parking performance on a moment to moment basis, we're assuming the system can achieve equilibrium

Assumptions

- System can achieve equilibrium
- Transaction data is representative of occupancy
- Drivers search uniformly (and legally)
- Price is only factor in parking demand
- Haven't assumed block-faces are probabilistically independent of one another
- No need to specify service-time distribution

Assumptions

- System can achieve equilibrium
- Transaction data is representative of occupancy
- Drivers search uniformly (and legally)
- Price is only factor in parking demand
- Haven't assumed block-faces are probabilistically independent of one another
- No need to specify service-time distribution
- Exogenous arrivals are Poisson



Future Work

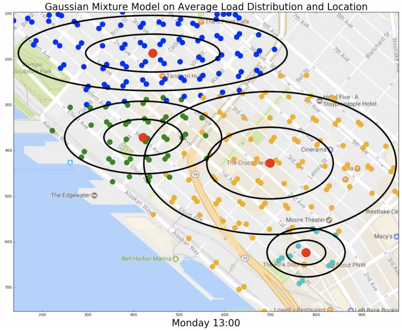
Open questions in parking research:

- Price discrimination due to:
  1. Garage/lot market power
  2. Maximum parking time
  3. Distance to popular destinations
- Effect of parking information systems on locational demand (decision to drive before leaving)
- Emerging effect of ride-sharing services—how will future curbside parking resources be most effectively utilized?

Future Work

How we're tackling these problems:

- Building a *structural* model *around* data that's currently available.
- Aiming to enable socially and politically actionable solutions to congestion



Credit: Tanner Fiez, UW EE

Concluding Remarks

- Black-box ML solutions may not be sufficient to adapt aging infrastructure and related policies to emerging technologies (distributed generation, autonomous vehicles)

Concluding Remarks

- Black-box ML solutions may not be sufficient to adapt aging infrastructure and related policies to emerging technologies (distributed generation, autonomous vehicles)
- We want to combine structural models from which control policy can be evaluated, with the naive data-analysis benefits of ML

Conclusion

Questions?

Data Sources

Data: IDAX, Seattle Dept of Transportation and `data.seattle.gov`

- block-face latitude/longitudes
- spaces per block (number of servers)
- curbside parking transactions since 2012 at each block-face (service times)
- traffic volume by time of day on select arterials (superset of drivers parking)

SDOT Data

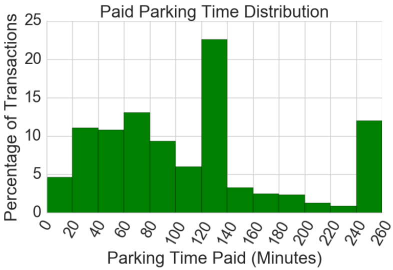


Figure 6: Distribution of transactions by paid parking time.

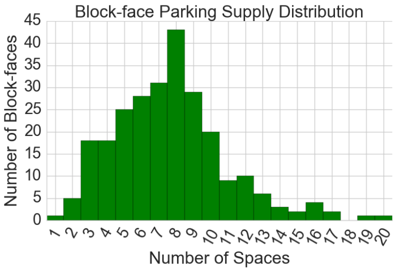
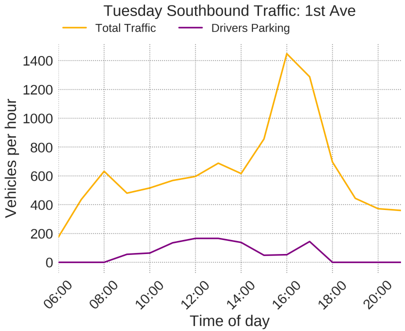
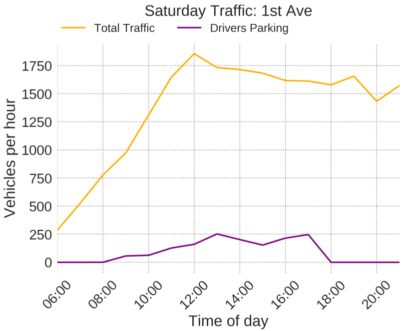


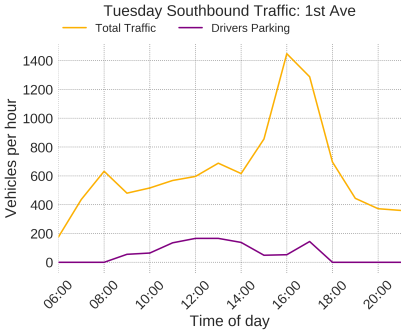
Figure 7: Distribution of parking spaces per block-face in Belltown.

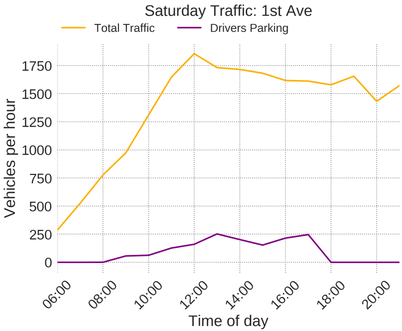
Proportion of Traffic Due to Parkers





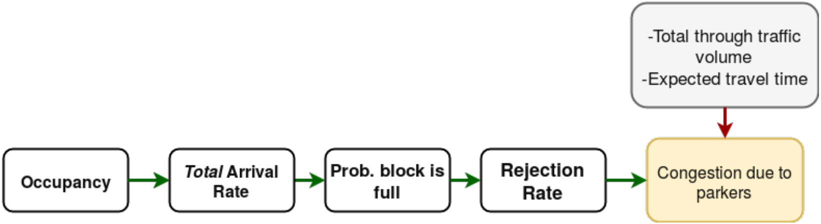
Proportion of Traffic Due to Parkers





What is the time-delay impact to through-traffic?

Calculating Congestion from Data



Congestion Caused by Parkers

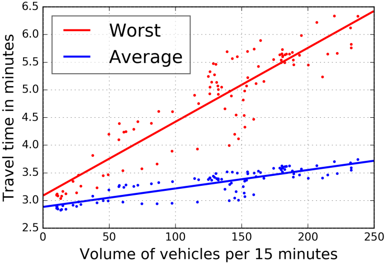


Figure 8: Estimates of travel time delay curve for measured volume versus historical delay

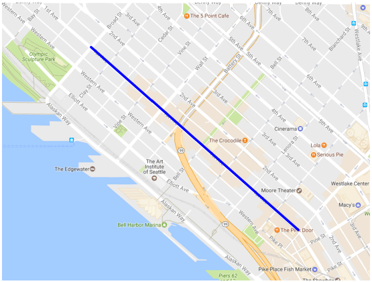


Figure 9: Belltown arterials with SDOT traffic volume data

Congestion Caused by Parkers

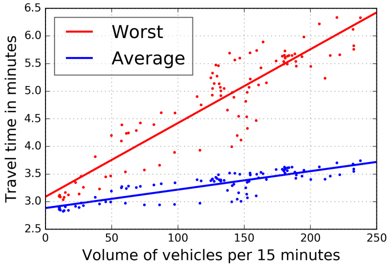


Figure 10: Estimates of travel time delay curve for measured volume versus historical delay

T : volume of cars $\rightarrow$ expected delay

Percent increase in delay:

$$\frac{T(N_{\text{total}})}{T(N_{\text{total}} - N_{\text{parking}})} - 1$$

Proof Sketch: Convexity of f

Let $x = ku$. Then we can think of (6) as

$$F(y,x) = (\tfrac{x}{k!} - \tfrac{1}{(k-1)!})y^k + \cdots + (\tfrac{x}{2!} - 1)y^2 + (x-1)y + x \quad (12)$$

$$y' = -D_x F \cdot (D_y F)^{-1} \quad (13)$$

and, by Quotient Rule:

$$y'' = \frac{D_x F \cdot (D_y^2 F \cdot y' + D_{x,y} F) - D_y F \cdot D_{y,x} F \cdot y'}{(D_y F)^2} \quad (14)$$

Proof Sketch: Convexity of f

Substituting in y' for the mixed partials, showing y'' boils down to showing

$$D_y^2 F \cdot y' + 2D_{y,x} F \geq 0 \tag{15}$$

Relying on the fact that (x, y) are a pair such that $F(x, y) = 0$, we get that

$$D_y^2 F \cdot y' + 2D_{y,x} F \geq y' F(x, y) = 0 \tag{16}$$

Proof Sketch: Convexity of f

We still need to show $y' > 0$.
By Gauss-Lucas (the roots of a polynomial are contained in the convex hull of the roots of its derivative), for fixed x all real parts of the roots of $D_y F$ are less than the root of $F(x, y)$. Since $D_y F \to -\infty$ as $y \to \infty$, at $F(x, y) = 0$. Recall we have that:

$$y' = -D_x F \cdot (D_y F)^{-1} \tag{17}$$

Since $D_y F \leq 0$ and since $D_x F > 0$, $y' > 0$

Future Work

What assumptions can we further address?

- Utilizing existing work on accurate estimation of occupancy from transaction data
- Incorporate factor analysis of location into parking demand/elasticity (hospital vs shopping mall)
- Simulate equillibrium in real downtown network and compare to numerical method
- Incorporate driver search behavior